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THESIS

DESIGN OF FAST EARTH-RETURN
TRAJECTORIES FROM A LUNAR BASE

by

Walter Anhorn

September, 1991

Thesis Advisor: Dr. Donald v. Z. Wadsworth

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The Apollo Lunar Program utilized efficient, i.e., Earth-return, transearth trajectories which employed parking orbits in order to minimize energy requirements. This thesis concentrates on a different type of transearth trajectory. These are direct-ascent, hyperbolic trajectories which omit the parking orbits in order to achieve short flight times to and from a future lunar base. The object of the thesis is the development of a three-dimensional transearth trajectory model and associated computer program for exploring trade-offs between flight-time and energy, given various mission constraints. The program also targets the Moon with hyperbolic trajectory, which can with a time-reversed trajectory; be used for targeting Earth impact points. The first-order model is based on an Earth-centered conic and a massless spherical Moon, using MathCAD version 3.0. This model is intended as the basis for future patched-conic formulation for the design of fast Earth-return trajectories. Applications include placing nuclear-deterrent arsenals on the Moon, various space support related activities and finally protection against Earth-threatening asteroids and comets using lunar bases.

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Design of Fast Earth-Return Trajectories from a Lunar Base

by

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ABSTRACT

The Apollo Lunar Program utilized efficient, i.e., Earth-return, transearth trajectories which employed parking orbits in order to minimize energy requirements. This thesis concentrates on a different type of transearth trajectory. These are direct-ascent, hyperbolic trajectories which omit the parking orbits in order to achieve short flight times to and from a future lunar base. The object of the thesis is the development of a three-dimensional transearth trajectory model and associated computer program for exploring trade-offs between flight-time and energy, given various mission constraints. The program also targets the Moon with a hyperbolic trajectory, which can with a time-reversed trajectory; be used for targeting Earth impact points. The first-order model is based on an Earth-centered conic and a massless spherical Moon, using MathCAD version 3.0. This model is intended as the basis for future patched-conic formulation for the design of fast Earth-return trajectories. Applications include placing nuclear-deterrent arsenals on the Moon, various space support related activities and finally protection against Earth-threatening asteroids and comets using lunar bases.

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TABLE OF SYMBOLS AND ABBREVIATIONS

This listing of symbols and abbreviations is to aid the reader in clarifying the multitude of symbols and abbreviations used in this thesis. They are in alphabetical order.

α_{E_ORB} : input value for posigrade or retrograde orbit

α_E : spacecraft right ascension at Earth launch

α_M : spacecraft right ascension at Moon intercept

α_{M1} : represents cosine of α_M

α_{M2} : represents sine of α_M

α_{EM} : represents the difference between α_M and α_E

α_{EM1} : represents the cosine of α_{EM}

α_{EM2} : represents the sine of α_{EM}

$\alpha_{E\Omega}$: represents the difference between the translunar trajectory longitude-of-ascending-node and spacecraft right ascension at Earth launch

$\alpha_{E\Omega 1}$: represents the cosine of $\alpha_{E\Omega}$

$\alpha_{E\Omega 2}$: represents the sine of $\alpha_{E\Omega}$

α_ι : represents an intermediate value to calculate the spacecraft right ascension on its trajectory

α_0 : represents spacecraft right ascension on its trajectory

α : represents the selenographic longitude for lunar surface impact

β_E : represents flight path azimuth at launch

β_{E1}	: represents cosine of β_E
β_{E2}	: represents sine of β_E
β_M	: represents flight path azimuth at moon intercept
β_{M1}	: represents cosine of β_M
β_{M2}	: represents sine of β_M
β_0	: represents flight path azimuth of the spacecraft on its trajectory
β_1	: represents cosine of β_0
β_2	: represents sine of β_0
C_E	: represents the sweep angle at launch
C_{E1}	: represents the cosine of C_E
C_{E2}	: represents the sine of C_E
C_M	: represents the sweep angle at moon intercept
C_0	: represents the sweep angle of the spacecraft on its trajectory
γ_E	: represents flight path angle at launch
γ_{E1}	: represents the cosine of γ_E
γ_{E2}	: represents the sine of γ_E
γ_M	: represents flight path angle at moon intercept
γ_{M1}	: represents the cosine of γ_M
γ_{M2}	: represents the sine of γ_M
γ_0	: represents the flight path angle of the spacecraft on its trajectory
γ_1	: represents the cosine of γ_0
γ_2	: represents the sine of γ_0
D	: represents the deflection angle

d_{asy} : represents the directrix-to-focus or axial offset of the asymptotic from the focus ; it is a variable valid for eccentricities greater than one

D_{M0} : represents the distance between the spacecraft and the lunar center at time t_0

δ_E : represents the spacecraft declination at earth launch point

δ_M : represents spacecraft declination at moon intercept

δ_0 : represents the spacecraft declination on its trajectory

δ : represents the selenographic latitude for surface impact on the moon

$\Delta\alpha$: see Figure 8

$\Delta\alpha_M$: represents intermediate value to calculate spacecraft right ascension on its trajectory

$\Delta\alpha_{M1}$: represents the cosine of $\Delta\alpha_M$

$\Delta\alpha_{M2}$: represents the sine of $\Delta\alpha_M$

$\Delta\alpha_0$: represents an intermediate value for the calculation of α_0

$\Delta\alpha_1$: represents the cosine of $\Delta\alpha_0$

$\Delta\alpha_2$: represents the sine of $\Delta\alpha_0$

$\Delta_{\rho 0}$: represents cosine of λ_N : see iterative process on page 12

Δ_{r0} : represents the difference between point-moon intercept flight time relative to perigee passage and the spacecraft flight time on its trajectory

eccen : represents eccentricity as an input value

$eccen_{min}$: represents the minimum eccentricity input value allowed

$energy_{min}$: represents the minimum specific energy required for launch

E_{h1} : represents the hyperbolic eccentric anomaly for earth launch

E_{h2} : represents the hyperbolic eccentric anomaly for point-moon intercept

E_{h3} : represents the hyperbolic eccentric anomaly for the spacecraft on its trajectory

E_{sh1} : represents the hyperbolic sine of E_{h1}

E_{sh2} : represents the hyperbolic sine of E_{h2}

E_{sh3} : represents the hyperbolic sine of E_{h3}

f_M : represents the input value for the true anomaly of the spacecraft at moon intercept

f_E : represents the true anomaly of the spacecraft at earth launch

f_{EM} : represents the difference of the true anomaly of the spacecraft at moon intercept and at earth launch

f_{M_MAX} : represents a constraint value on f_M if required to be used

f_{M_MAX1} : represents the cosine of f_{M_MAX}

f_{M_MAX2} : represents the sine of f_{M_MAX}

f_{M1} : represents the cosine of f_M

f_{M2} : represents the sine of f_M

f_0 : represents the true anomaly of the spacecraft on its trajectory

f_{M0} : represents the cosine of f_0

θ : represents the first Euler angle

h_E : represents the geocentric altitude of launch point

i_M : represents the inclination of the moon's orbit plane

i : represents the translunar trajectory inclination

i_1 : represents the cosine of i

i_2 : represents the sine of i

k_E : represents the earth's gravitational constant

k_M : represents the moon's gravitational constant

r_E : represents the earth's mean radius

r_M : represents the moon's mean radius

R_{E1} : represents the spacecraft radial distance at earth launch

R_{M1} : represents the spacecraft radial distance at moon intercept

r : represents the radial distance ratio

R_a : represents apogee distance : valid only for eccentricities less than 1

R_p : represents the perigee distance

R_0 : represents the input value for the position of the spacecraft on its trajectory

$R_{M'}$: represents the radial velocity at point-moon intercept

R_x : represents an intermediate value to calculate f_0

ρ_{EM} : represents the Earth-Moon mean distance

$sign_{fEM}$: represents an intermediate value calculated to insure correct use of sign when calculating α_{EM}

S_{fEM} : represents the input value of +1 or - 1 to insure correct sign usage in calculating α_{EM}

T_p : represents a circular orbit period for radius R_p

t_E : represents Earth launch flight time relative to perigee passage

t_{E1} : represents an intermediate value used to calculate t_E

t_M : represents the point-moon intercept flight time relative to perigee passage

t_{M1} : represents an intermediate value used to calculate t_M

t_{EM} : represents the point-moon translunar trajectory flight time

t_{M0} : represents an intermediate value used to calculate t_0

t_0 : represents the flight time of the spacecraft on its trajectory

V_p : represents the perigee velocity

V_E : represents the spacecraft velocity at launch

V_M : represents the spacecraft velocity at moon intercept

V_0 : represents the spacecraft velocity on its trajectory

ϕ_M : represents the input value of the lunar sweep angle at moon intercept

ϕ_{M1} : represents the cosine of ϕ_M

ϕ_{M2} : represents the sine of ϕ_M

ϕ_{M0} : represents the difference between the lunar sweep angle at moon intercept and the moon's mean orbital rotation rate multiplied by $\Delta_{\tau 0}$

ϕ : represents the first Euler angle

X_M : represents the X-coordinate for the rectangular coordinates of the moon at point-moon intercept

$X_{M'}$: represents the X-component for calculating radial velocity at point-moon intercept

X_0 : represents the X-coordinate for the rectangular coordinates of the spacecraft on its trajectory

X_{M0} : represents the X-coordinate for the center of the moon

X : represents the difference between X_M and X_{M0}

X_{lun} : represents the X-coordinate for the rectangular selenographic coordinates

Y_M : represents the Y-coordinate for the rectangular coordinates of the moon at point-moon intercept

$Y_{M'}$: represents the Y-component for calculating radial velocity at point-moon intercept

Y_0 : represents the Y-coordinate for the rectangular coordinates of the spacecraft on its trajectory

Y_{M0} : represents the Y-coordinate for the center of the moon

Y : represents the difference between Y_M and Y_{M0}

Y_{lun} : represents the Y-coordinate for the rectangular selenographic coordinates

Ψ_M : represents the third Euler angle

Z_M : represents the Z-coordinate for the rectangular coordinates of the moon at point-moon intercept

$Z_{M'}$: represents the Z-component for calculating radial velocity at point-moon intercept

Z_{M0} : represents the Z-coordinate for the center of the moon

Z : represents the difference between Z_M and Z_{M0}

Z_{lun} : represents the Z-coordinate for the rectangular selenographic coordinates

Ω : represents the translunar trajectory longitude-of-the-ascending-node

ω_E	: represents the Earth's angular rotational rate
ω_M	: represents the Moon's mean orbital rotation rate
ω	: translunar trajectory argument-of-perigee

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I. INTRODUCTION

This chapter provides a general description of this thesis and is divided into two sections. The first section presents background information pertaining to translunar trajectory design. The second section delineates the objectives and limitations of this project.

A. BACKGROUND

While the impending mission of establishing a permanent lunar base stands in our future, it is appropriate to study this type of mission by investigating various translunar trajectories. The trade-off between flight time and energy required must be determined prior to considering the type of translunar trajectory to design. The energy required for the hyperbolic trajectory can then be compared to that of the Apollo mission to determine the increase in energy and thereby cost increases.

The Apollo Lunar Program of the 1960s utilized energy-efficient translunar trajectories. This thesis will analyze a different class of trajectories named fast Earth-return trajectories. These trajectories utilize a direct-ascent from the lunar surface and omit the lunar parking orbit used in the Apollo missions. These direct-ascent trajectories differ considerably from translunar trajectories previously considered and discussed in this thesis. (Wadsworth, 1991)

B. OBJECTIVES AND LIMITATIONS

The objective of this thesis is to develop a trajectory model and associated computer program for systems studies of fast Earth-return trajectories from a lunar base. The scope of the project involves a step process which evolves in complexity, as the design moves from a massless point-Moon, to a massless spherical Moon intercept solution. Finally, the groundwork is laid for a patched-conic model which culminates in a solution for intercept on a Moon with finite lunar mass. A novel feature is the use of a time-reversed trajectory for targeting Earth impact points. In other words, the lunar impact point serves as the lunar launch point for a time-reversed trajectory. The complete patched conic approximation, as discussed in Chapter II, is not developed in the program due to scope of work and time constraints.

II. ANALYSIS APPROACH

This chapter sets the foundation of the thesis by delineating a flow chart of tasks that are required to be accomplished to reach the objective of the thesis. A flow chart represented in Figure 1, summarizes the work to be accomplished. The last block of the diagram represents further research work.

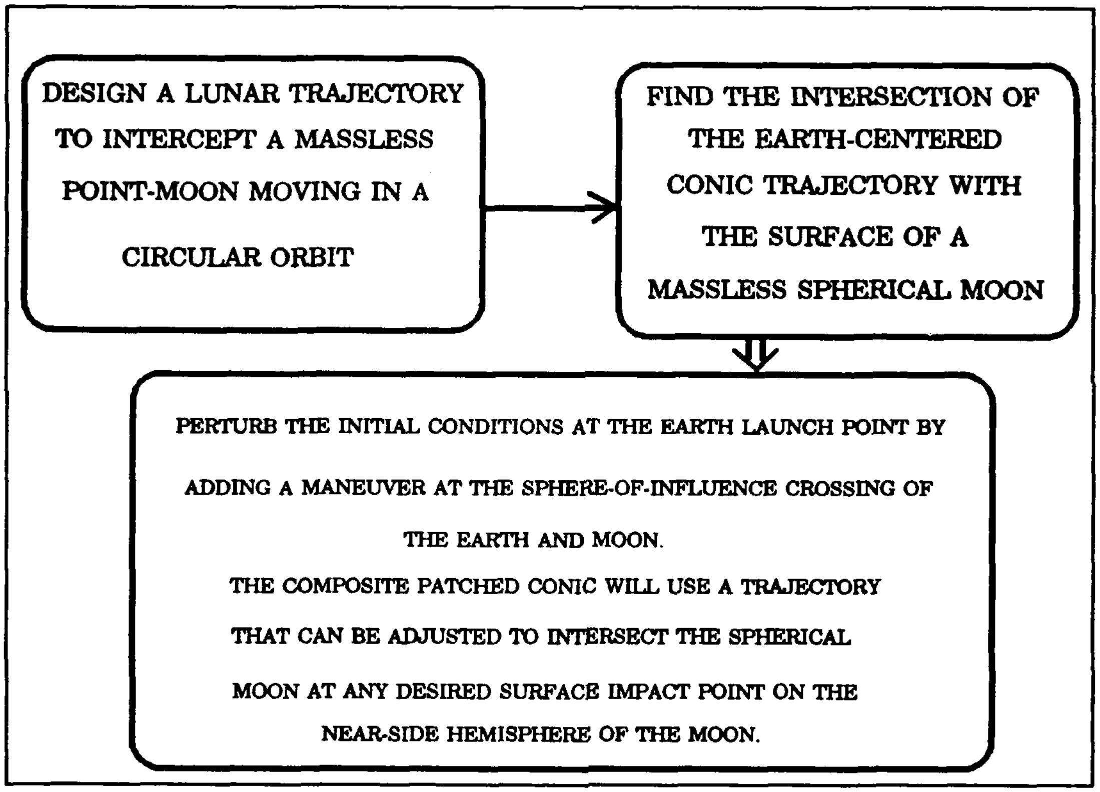

Figure 1. Translunar Trajectory Design Flowchart.

Precision translunar/transearth trajectory design requires time-consuming, step-by-step numerical solution of the exact differential equations of motion, including lunar and solar ephemerides. For preliminary design, the computation time can be reduced a

hundred-fold or more by employing a closed-form "patched-conic" approximation to the precision trajectory. Since the patched-conic is a composite of the two conics, it can be formulated explicitly in terms of elementary functions of the key parameters. This provides better insight into design trade-offs than numerical computation. The patched conic approximates the gravitational effects of the Earth-Moon system. It ignores the small perturbation due to the Sun.

Two approaches were considered for developing the patched-conic model. Both approaches employ Earth-centered and Moon-centered coordinate frames with respect to which the conic trajectories are formulated. Although the conic trajectories are planar with respect to the Earth-centered and Moon-centered frames, they form three-dimensional curves as viewed in an inertial barycentric frame. In the first approach, the coordinate frames are non-inertial, being attached to the Earth and Moon which travel in (idealized) circular orbits (with respect to the Earth-Moon inertial frame). The second approach employs non-rotating, inertial Earth-centered and Moon-centered coordinate frames. These frames are attached, respectively, to a fictitious Earth and fictitious Moon which travel at constant velocity with respect to the barycentric frame. The implications of the second approach are briefly stated in Appendix A. The advantage of the first approach, pursued in this thesis research, is described in section A. (Wadsworth, 1991)

A. TRANSLUNAR TRAJECTORY DESIGN

The first consideration given to designing a translunar trajectory is to decide on what translunar trajectory types will be used. Since this thesis concentrates on fast Earth-

return trajectories, eccentricities greater than one will be utilized. The reason for using non-inertial coordinate frames is discussed in section A.2.

1. Translunar Trajectory Types

Figure 2 illustrates all possible trajectory types that can be considered for Earth-Moon trajectory design. The radius of the Earth (R_E), radius of the Moon (R_M), spacecraft true anomaly at launch (f_E), and spacecraft true anomaly at moon intercept (f_M) are projected in Figure 2.

Since only trajectories with eccentricities greater than 1.0 are being considered, only type I and III hyperbolic trajectories will be used in thesis research (see Table I).

2. Non-Inertial Coordinate Frames

The Earth-centered and Moon-centered coordinate frames are non-inertial frames, since they are in a state of gravitational free-fall about the Earth-Moon barycenter (center of gravity). As a consequence, the composite patched-conic provides a better approximation to the restricted, three-body spacecraft motion than would be suggested by simply joining two conics.

Within the Moon's region-of-influence, the terrestrial perturbing acceleration of the spacecraft is nearly the same as the centripetal acceleration of the Moon-centered coordinate frame. In this frame, these accelerations nearly cancel; this can be deduced from the facts that the distances of the spacecraft to the Earth and the Moon to the barycenter are comparable and the mass of the Earth is comparable to the equivalent mass at the barycenter. For the Earth-centered phase, there is a similar benefit over

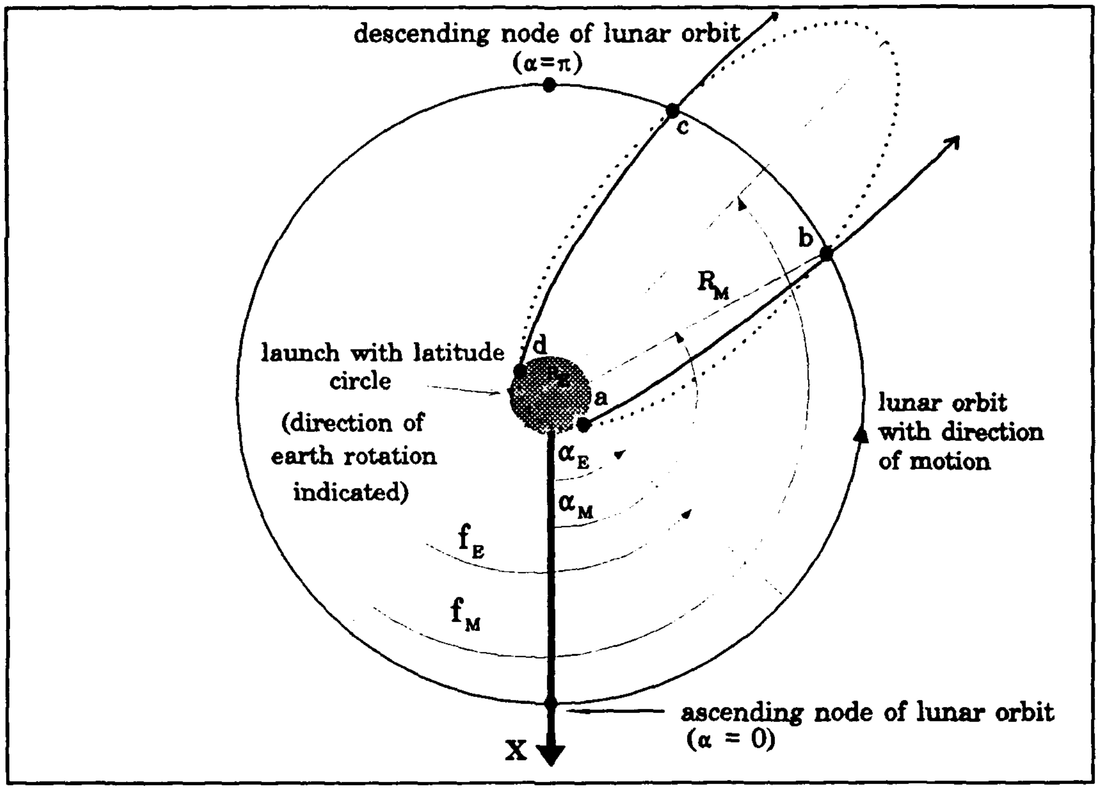

Figure 2. Equatorial plane projection.

<u>TRANSLUNAR TRAJECTORY TYPES:</u>

TYPE I posigrade : arc ab

TYPE I retrograde : arc dc

TYPE II posigrade : arc abc (e < 1)

TYPE II retrograde : arc dcb (e < 1)

TYPE III posigrade : arc dab

TYPE III retrograde : arc adc

TYPE IV posigrade : arc dabc (e < 1)

TYPE IV retograde : arc adcb (e < 1)

Table I. Trajectory Types.

most of the trajectory, especially for parabolic and hyperbolic trajectories. This cancellation does not occur in the case of patched-conics based on inertial frames as described in Appendix A (Wadsworth, 1965).

The Earth-centered translunar conic trajectory may be either elliptic or hyperbolic. The latter corresponds to shorter flight times and is the only case that needs consideration for fast trajectories. For the same reason, the moon-centered conic approach trajectory is also modeled as hyperbolic in this study. Both the Earth-centered and Moon-centered frames are non-rotating. The motions of the Earth and Moon about their common barycenter are approximated by coplanar circular orbits. In general, the plane of these circular orbits does not coincide with the three dimensional trajectory of the spacecraft. (Wadsworth, 1991)

B. SOLUTION FOR POINT-MOON INTERCEPT

The least complicated form of a first order solution is for a point-moon intercept. Figure 3 illustrates the relevant parameters of the problem. Appendix B, utilizing MathCAD 3.0, numerically solves the first order solution of point-Moon and massless spherical Moon intercept. In Figure 3 the following assumptions are made:

- The Earth-centered cartesian coordinate frame is non-rotating (indicated by X,Y,and Z).
- The Moon-centered cartesian coordinate frame is non-rotating (indicated by x,y,and z with the x-axis parallel to the X-axis, and the z-axis tilted at angle i_M for the Z-axis.)

The initial point-Moon intercept solution represents a simple three-dimensional model; three-dimensional considerations are incorporated in the first-order solution solved in Appendix B. These considerations will be discussed in section B.2.

1. Three Dimensional Point-Moon Model

Figure 3 illustrates the three-dimensional model of the first-order solution at point-moon intercept while Figure 4 further illustrates aspects of the three-dimensional model. Figure 5 delineates a flowchart of tasks to be accomplished in the three-dimensional model of the first-order solution of point-Moon and massless spherical Moon intercepts. This flowchart summarizes steps used to construct Appendix B. Symbols and abbreviations are defined in the Table of Symbols and Abbreviations. The hyperbolic trajectory yielded an energy requirement of 63.2 km^2/sec^2 while the Apollo mission required a minimum energy requirement of 61.8 km^2/sec^2. This results in a 2% increase in energy by using the hyperbolic trajectory, and as a consequence, will increase launch costs. Considering that the time of flight is reduced by more than 50%, the 2% energy increase penalty seems feasible for this mission.

2. Three Dimensional Considerations

Figure 6 illustrates the relationship of orbit eccentricity and spacecraft true anomaly at moon intercept. Figure 7 is an exploded view of Figure 6 for user interpretation which shows eccentricities between 1.0 and 2.0. These diagrams must be referenced by the user to insure appropriate inputs for solution in Appendix B. Figure 8 illustrates a cone sweeping out a circular area of the Earth. The Earth surface within the cone defines a zone of inaccessibility for Earth launch sites, given the specified true

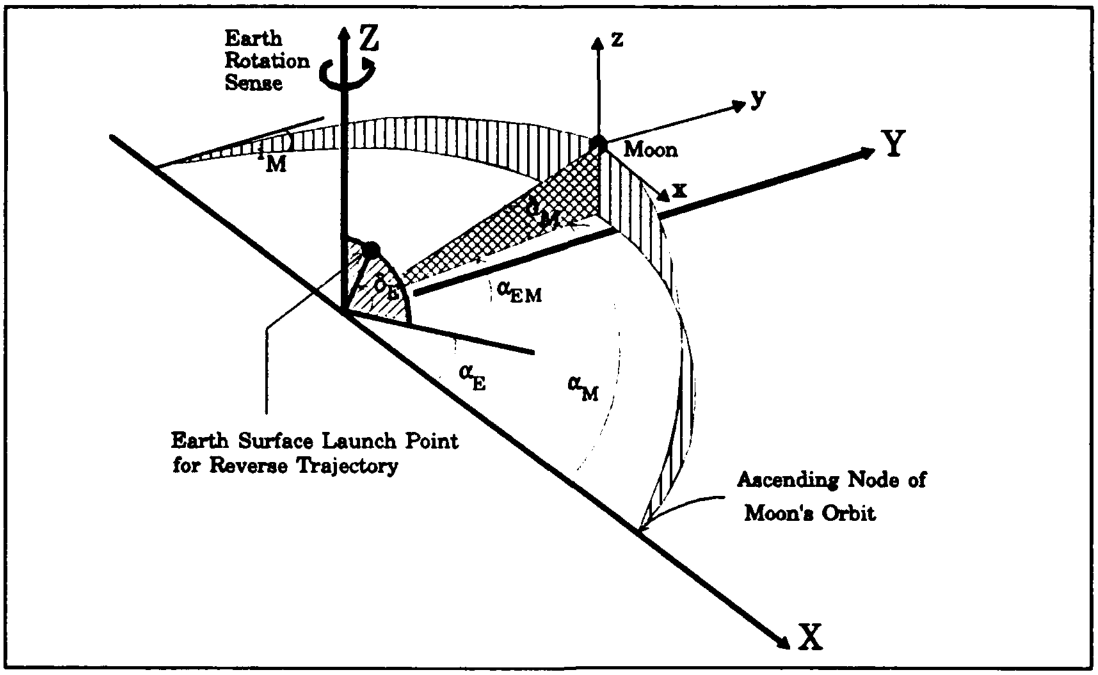

Figure 3. Three dimensional point moon model.

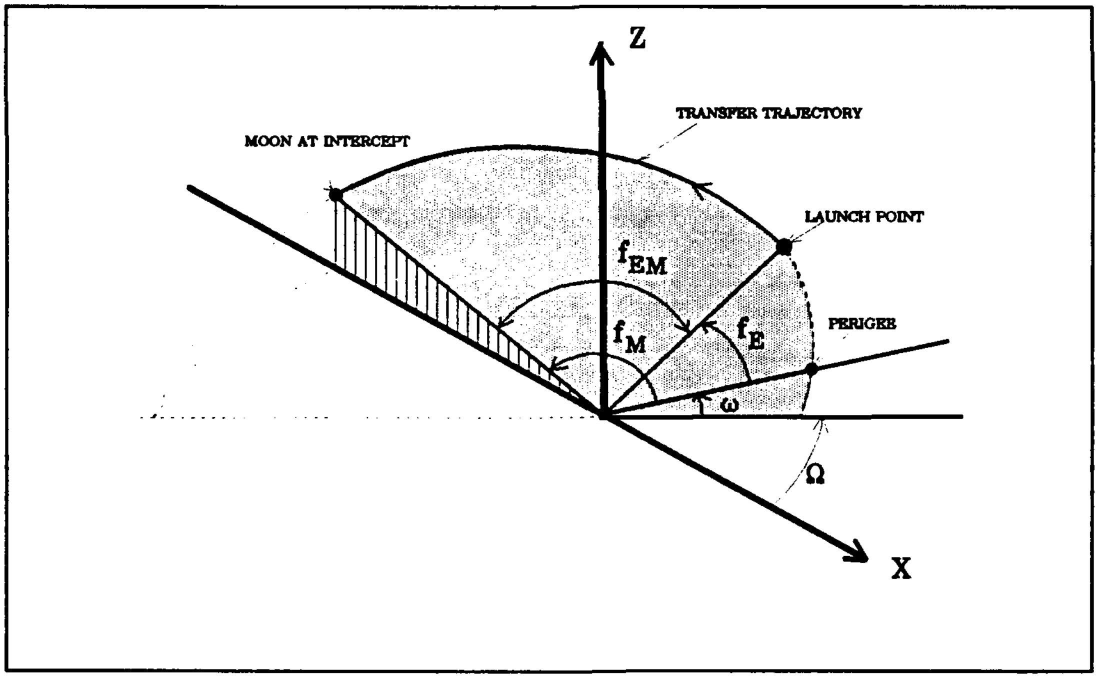

Figure 4. Three dimensional model focusing on the point Moon.

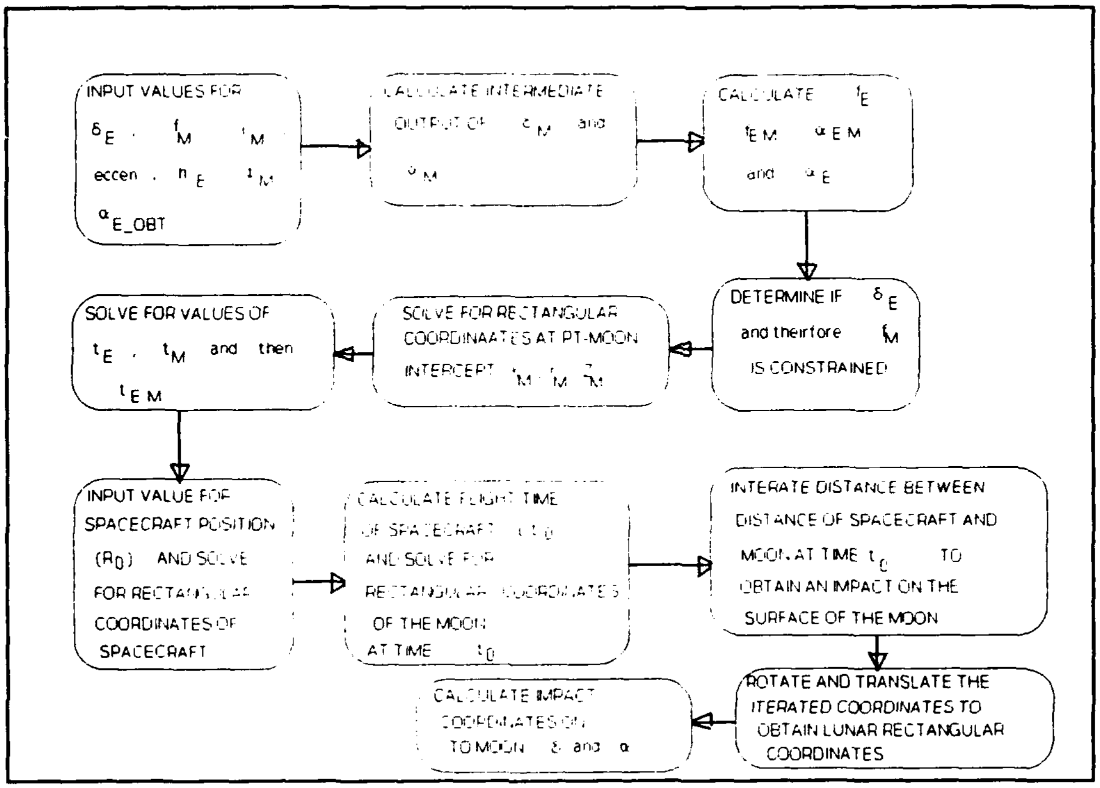

Figure 5. Translunar Trajectory Program Flowchart.

anomaly at moon intercept. This fact constrains inputs to the program in Appendix B. If the launch site is found to be in the restricted cone region a different trajectory must be used.

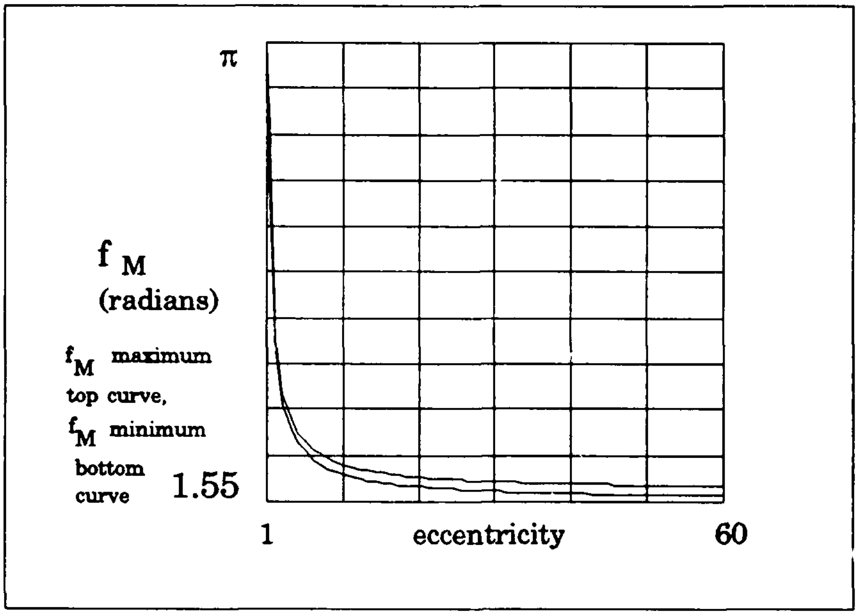

Figure 6. Eccentricity versus maximum and minimum true anomaly at Moon intercept.

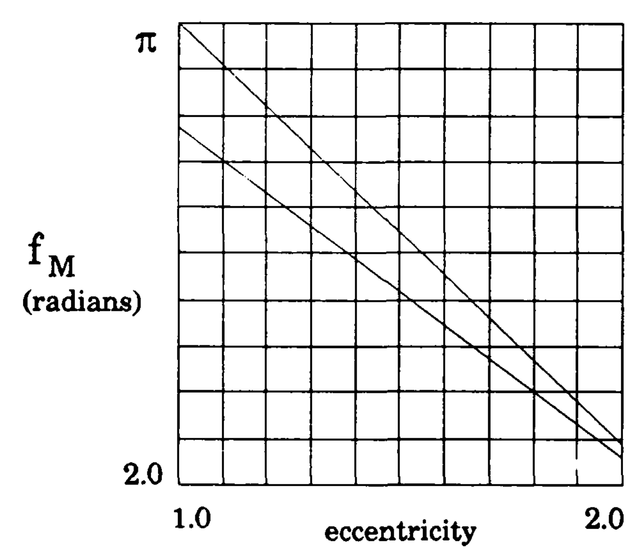

Figure 7. Exploded view of Figure 6.

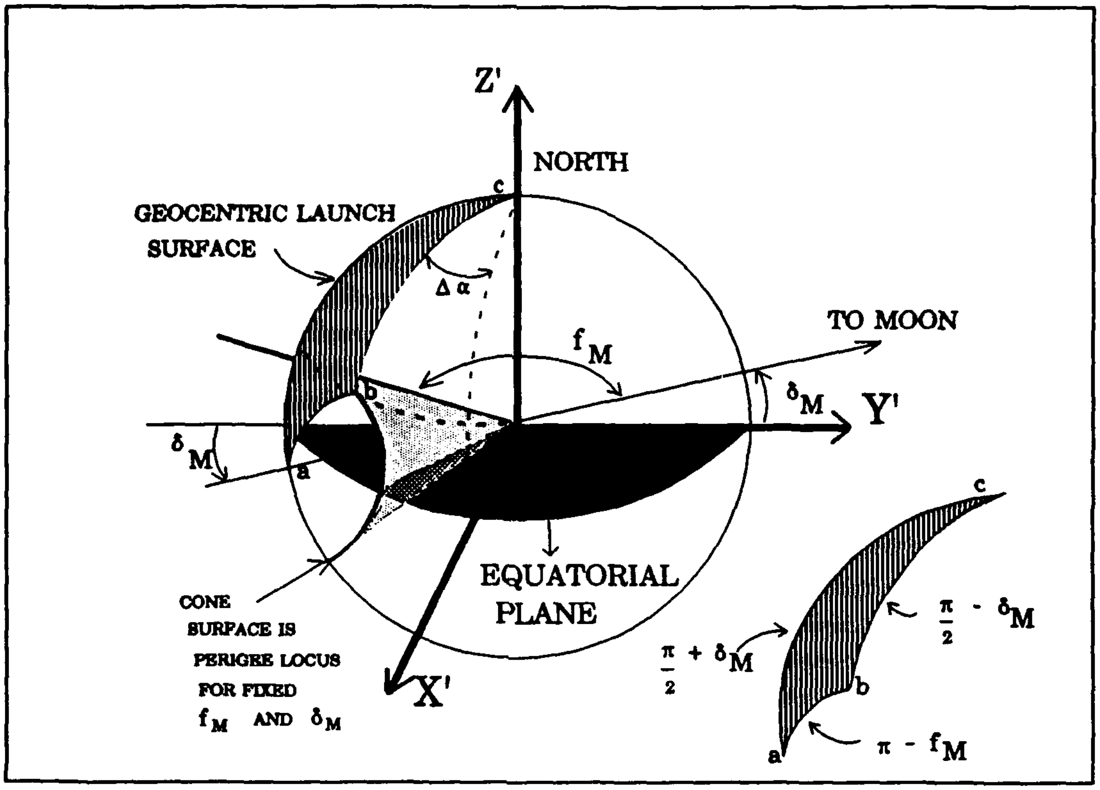

Figure 8. Three dimensional considerations.

C. SOLUTION FOR MASSLESS SPHERICAL MOON INTERCEPT

The final solution in Appendix B, is the massless spherical Moon intercept. An iterative technique is employed to obtain an impact on the lunar surface. This technique is illustrated and described in section C.1. Selenographic coordinates are utilized to obtain the solution for the massless spherical moon intercept and are described in section C.2.

1. Iterative Technique for Lunar Surface Impact

Figure 9 illustrates the method to ascertain the lunar surface impact point iteratively. The iterative process is defined in the following steps:

- Determine point-moon intercept coordinates ($R = \rho_{EM}$, f_M, ϕ_M, and t_M are inputs).

- Select trial value of radial distance : $R_0 \approx \rho_{EM}$-D.

- Calculate the corresponding flight time from perigee, t_0, and separation flight time D_0.

- Find the next iteration : $R_1 = R_0 + (D_0 - D)\cos\lambda_0$.

- $\mathbf{R_n} \cdot \mathbf{D_n} \div R_n D_n = \cos\lambda_n$ and $R_n = R_{n-1} + (D_{n-1} - D)\cos\lambda_{n-1}$ (bold print represents vector quantities).

- Continue until n = 10 or $|D_{n-1} - D| < 1$ km.

This iterative technique is employed in Appendix B, however, solutions are calculated manually by the program user without a terminal loop.

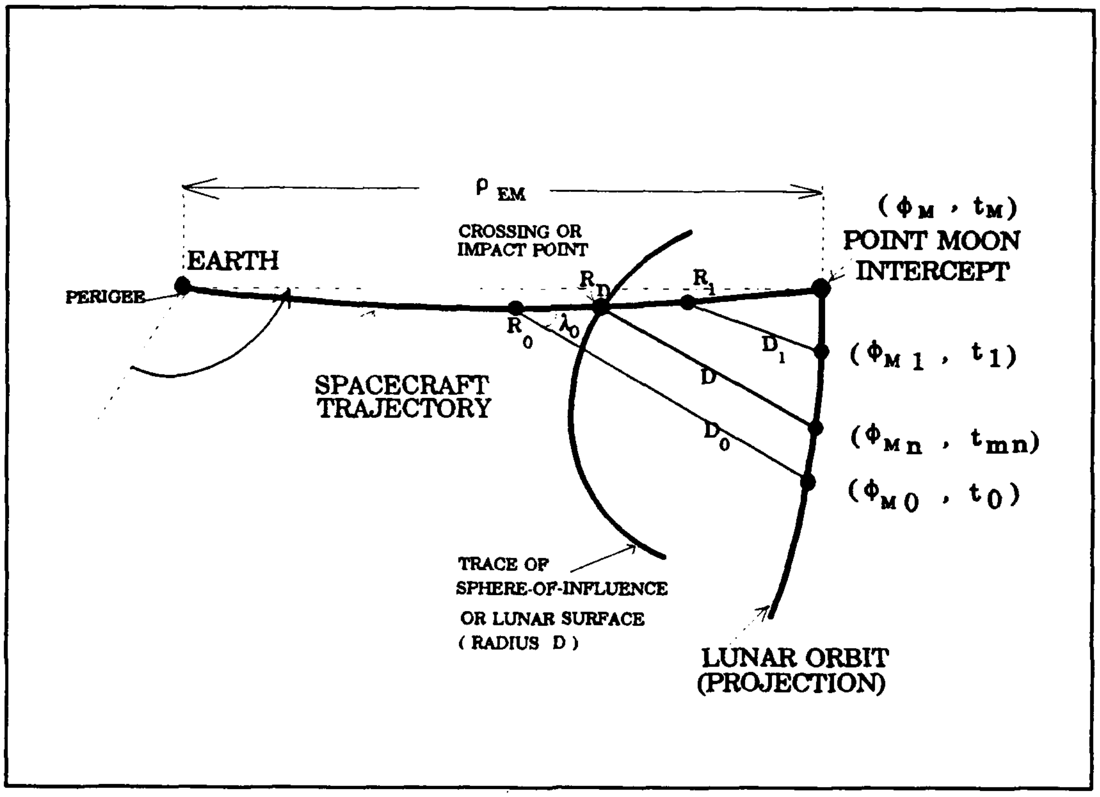

Figure 9. Iterative technique model.

2. Selenographic Coordinates

Impact points on the near-side of the moon are expressed in selenographic coordinates, for which the zero degree longitude and latitude points at the Earth. The North, South, East, and West directions indicated on Figure 13 for latitude and longitude, are for an observer on the Earth. The lunar librations (Roy, 1988) are neglected in this definition.

a. Euler angles

Figures 10 and 11 express Euler angle transformations which are incorporated in Appendix B to relate selenographic coordinates to Earth-centered coordinate systems. The first Euler angle,(ϕ), is zero because this angle is not required in this analysis. The second Euler angle rotation,(θ), is the angle i_M in Appendix B. The third Euler angle rotation,(Ψ_M), is the angle ϕ_M in Appendix B. (Goldstein, 1951)

b. Lunar Impact

Figure 12 illustrates a selenographic coordinate solution for lunar impact of the spacecraft. Since time reversal is employed in Appendix B, the Moon moves backwards in its orbit for the translunar trajectory. The sign on ω_M is reversed in Appendix B for this reason.

The selenographic latitude for surface impact is designated δ and the selenographic longitude for surface impact is designated α. Figure 13 illustrates these latitudes, longitudes and their signs. The solution for α has a factor of π added to it to insure that the positive X-axis points to the Earth. Figure 14 displays a locus of impact points for varying time offsets (t_{off}). The axes units are in radians. Varying t_{off}

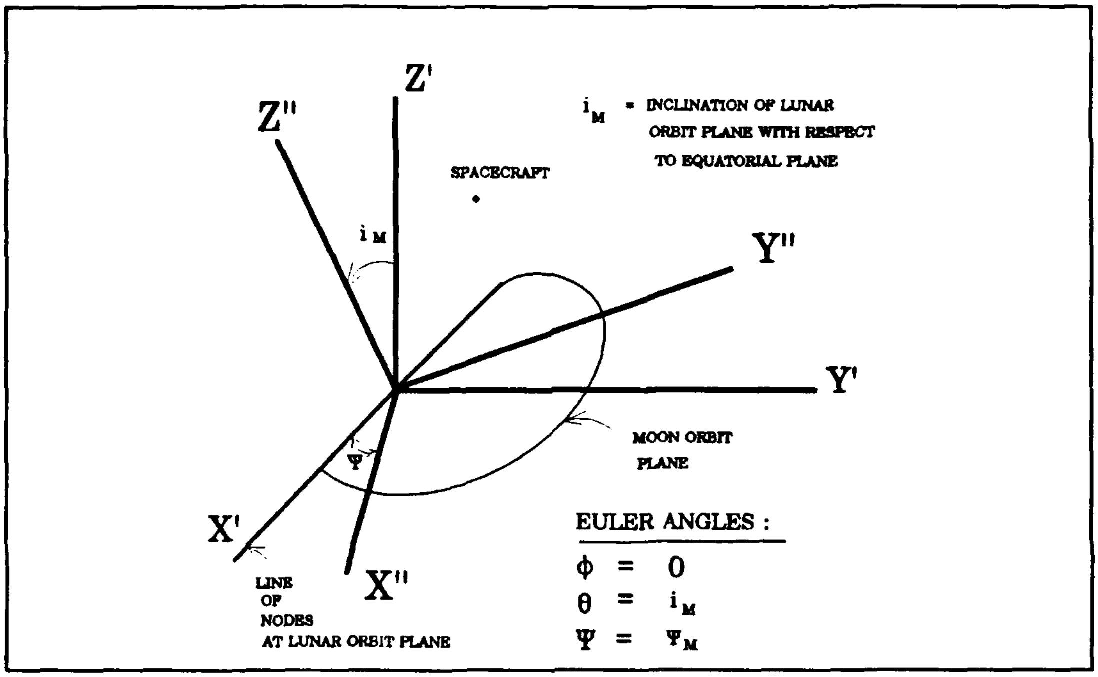

Figure 10. Second Euler angle rotation.

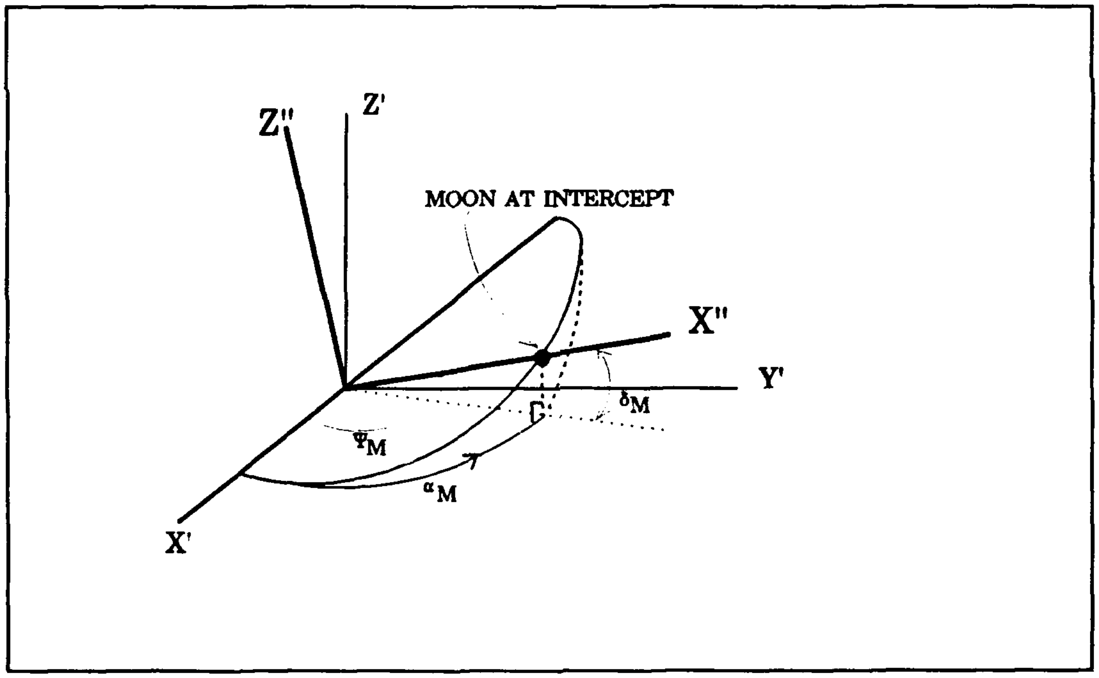

Figure 11. Third Euler angle rotation.

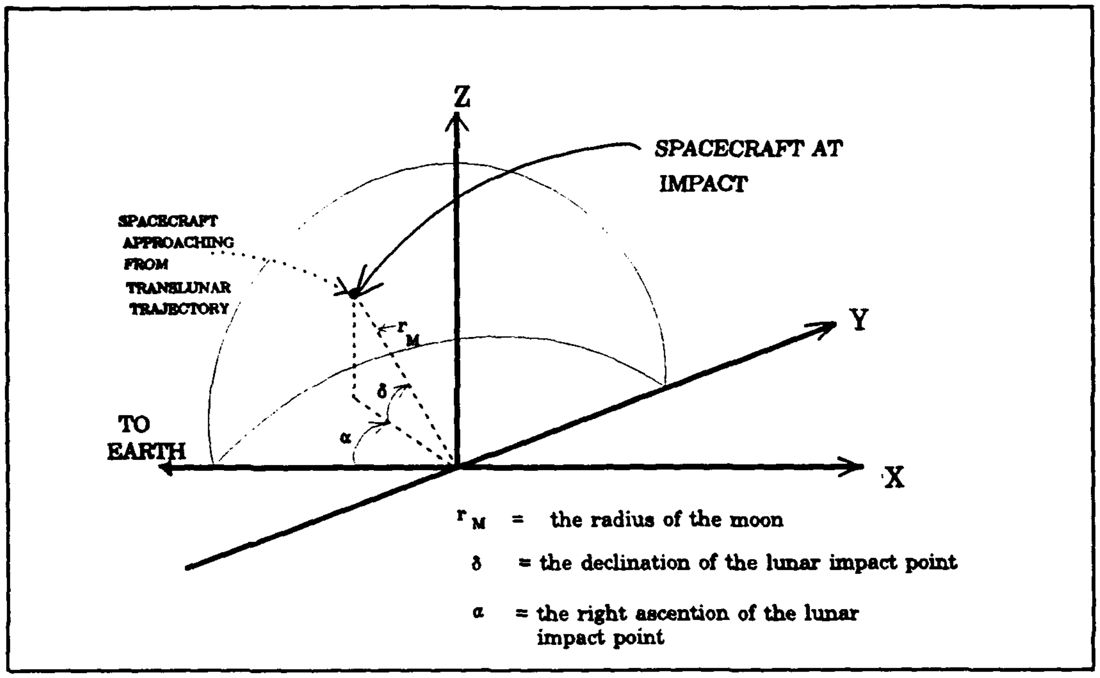

Figure 12. Lunar centered frame.

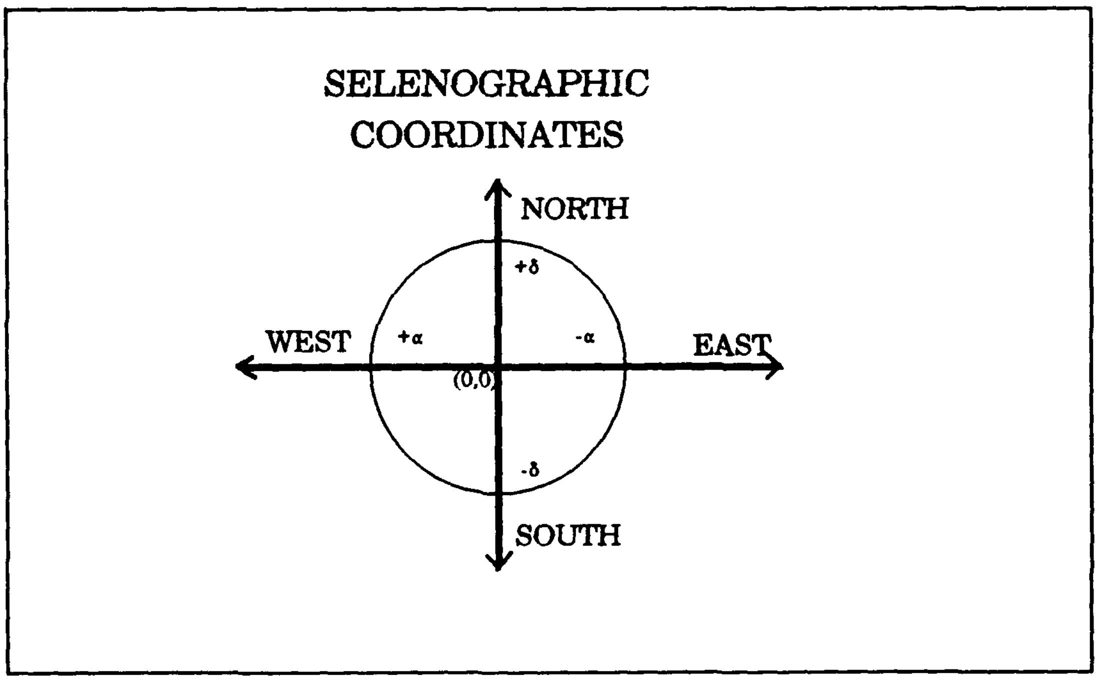

Figure 13. Selenographic coordinates.

is equivalent to varying the input value of the lunar sweep angle (ϕ_M). By providing an offset from the point-Moon intercept declination (δ_{M_off}), the user can target from North to South (see Figure 13). This factor can be combined with (t_{off}) to produce an arbitrary location of the impact point on the lunar near side.

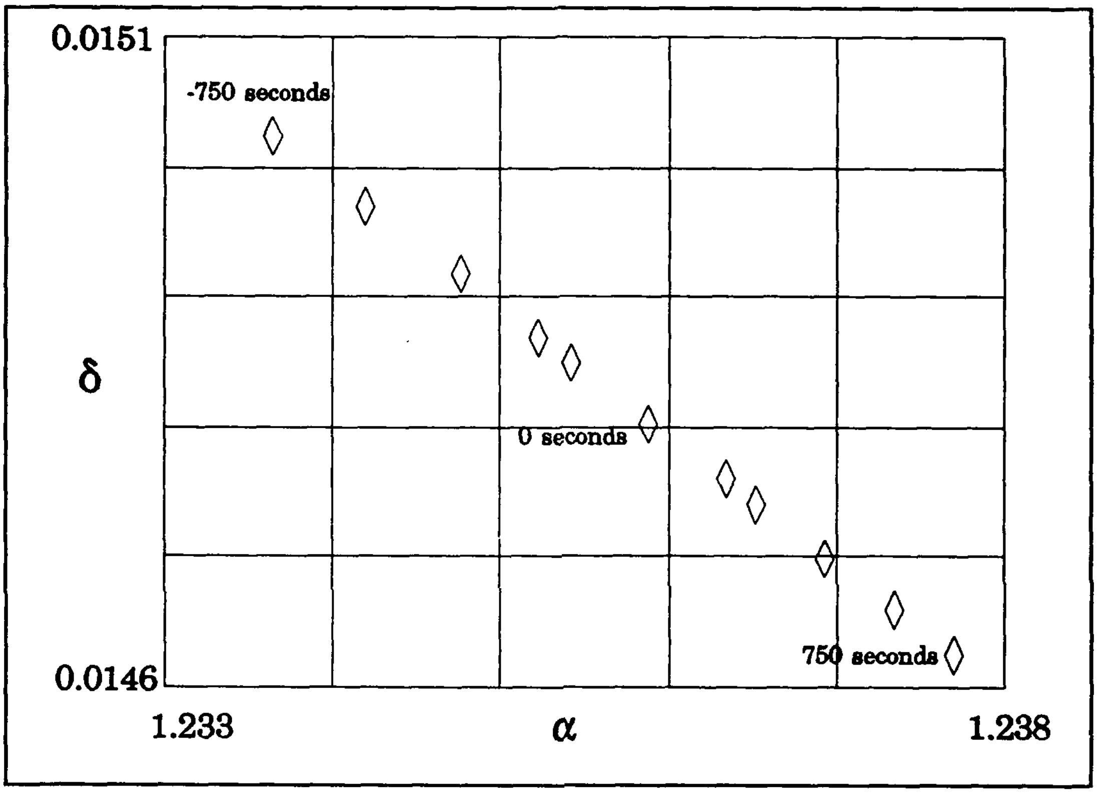

Figure 14. Locus of lunar impact points.

D. PATCHED CONIC APPROXIMATION

The method of calculation for the massless spherical moon intercept model can also be used for sphere-of-influence calculations if the radius of the sphere-of-influence is substituted for the radius of the Moon. The patched conic lunar impact point, using a

finite lunar mass model, can then be approximated by an iteration process identical to the process used in Appendix B (Bate et al., 1971).

From the final patched conic iteration, the impact point for the Moon-centered conic trajectory can be calculated. This calculation is made using the same method as Appendix B.

III. APPLICATIONS

Fast Earth-return trajectories have numerous applications, such as for requirements in evacuating a lunar base in an emergency or the establishment of a lunar-based strategic deterrent force. This chapter will discuss such applications, their development, and technical feasibility.

A. STRATEGIC PEACE INITIATIVE CONCEPT

The ABM treaty prohibits nuclear weapons in space. If we look at a future world, this treaty might possibly be altered or not exist. The Strategic Peace Initiative is a hypothetical plan to place nuclear weapons on the Moon and to eliminate them from the Earth. The reasoning for this replacement is to prolong the strike capability and reaction time that now exists, thereby increasing stability.

1. Background Information

This section describes a hypothetical Strategic Peace Initiative, a plan to eliminate the risk of global nuclear war and simultaneously promote peaceful uses of space. The SPI concept could be a step toward reducing present nuclear confrontation as well as "denuclearizing" the Earth. (Wadsworth, 1988)

Rather than using our current posture of SDI, the SPI concept can encourage peaceful exploitation of space (Gray, 1985). SPI resources could provide the logistics support for a new era of nonmilitary space activities, such as manned space exploration, servicing geostationary communication satellites, safe disposal of radioactive waste, clean-

up of hazardous space-debris, and eventually, deflection of Earth-threatening asteroids or comets.

Although the basing of deterrent strike forces in space, particularly on the Moon, has already been proposed; this proposal is quite unique. The novelty of an SPI concept is that by a new treaty, the United States and Soviet Union would share the "high ground" of space. As with both the INF and Start treaties, success of the SPI treaty would depend on whether it serves the mutual interests of parties involved. (Wadsworth, 1988)

The free world's offensive deterrent policies have prevented a global nuclear confrontation to date. Unfortunately, this policy is dangerously unstable because of the critically short decision time for launching a counterstrike to a perceived nuclear attack. SPI would eliminate or greatly reduce the danger of accidental nuclear war by increasing decision time from minutes, in the case of an SLBM attack, to a more comfortable two days.

The primary justification for SPI is survival in a nuclear age, consequently the emphasis in this proposal is on deterrence. A secondary, economically-compelling justification is that much of the SPI investment in global security could also serve the civilian space program. They both require similar logistics elements:

- A heavy-lift launch vehicle
- A space tug (space/lunar transfer vehicle)
- Earth/Moon space communications and tracking networks
- A manned lunar base

2. SPI Architecture

The minimum-energy transearth strike trajectory has a 4.7 day flight time. Doubling the injection energy yields a three day trajectory. High-energy strike trajectories, taking two days or less, still permit nuclear strike weapons to escape the Earth-Moon system if aborted. Appendix B provides such a time of flight (approximately 1.9 days). Shortening the flight time from 4.7 days to 2 days results in only a two percent increase in transearth trajectory energy, which is equivalent to providing the additional velocity increment typical of an ICBM. Atmospheric re-entry velocity does not increase significantly between 4.7 day and 2 day trajectories, which simplifies re-entry vehicle design. To minimize vulnerability to intercept, the trajectories must be of the direct-ascent type without lunar parking orbits, in contrast to the more economical supply shuttle trajectories which must also be utilized. (Wadsworth, 1988)

The SPI communications networks and surveillance systems would preferably be located at the natural synodic satellite points, designated L1 through L5, which are stationary with respect to the Earth-Moon system and would minimize station-keeping fuel. These points are therefore an ideal location for placing these communications satellite systems. (Farquar, 1970)

The two opposing lunar base zones, each approximately one hundred kilometers in diameter would contain the strike force missile silos and manned command/support complexes. These zones would be located on approximately opposite sides of the moon. They might occupy polar sites to take advantage of full-time sunlight

for solar power. Determining optimum zone locations requires analysis if the optimum strike trajectory missions for both bases. (Wadsworth, 1988)

Similar to the deployment of the stages of SDI, SPI would be implemented in stages and be subject to treaty compliance verification (possibly a United Nations role) at each stage. Both superpowers would deploy the following five stages:

- NON-NUCLEAR DEPLOYMENT : Lunar bases, communications, and surveillance systems deployed.
- FLIGHT TESTS : Test strike force with dummy warheads to demonstrate accuracy and reliability.
- NUCLEAR DEPLOYMENT : Equip lunar bases with nuclear missiles and phase out Earth-based strategic nuclear forces.
- VERIFICATION : Surveillance of lunar bases and Earth-Moon space; inspection and monitoring of Earth-based nuclear device manufacturing and test facilities; and inspection of translunar cargos.
- STRATEGIC REDUCTION EVOLUTION : Bilateral reduction of strategic nuclear forces to minimum required for asteroid deflections; increase in cooperative nonmilitary space ventures by utilization of lunar bases.

B. SPI TECHNICAL FEASIBILITY

Initial deployment of SPI relies on technology that has or will soon be proven by other programs. In fact, SPI can virtually be used as a spin-off of developed SDI technology. Programs such as TDRSS, NASA space station, OMV, NASP, HEDI, and the APOLLO lunar programs are examples of some of these programs.

Challenges in technology do exist. These challenges include a precision guided re-entry vehicle, developing "smart" surveillance techniques, and prevention of adverse

physiological effects of long-term residency in low gravity. At an interval of deployment of ten to twenty years, the fully deployed SPI deterrent force system probably would cost comparably to the proposed SDI system. (Wadsworth, 1988)

Under the SPI treaty, a sovereign lunar base zone would be assigned to each superpower for basing its nuclear strike force. These zones would be located on approximately opposite sides of the moon. The optimum size and location of the zones would be a compromise among the following six basic requirements:

- Minimize Surveillance Cost: Requires minimizing zone size; locate bases on the Moon's visible side to take advantage of surveillance redundancy provided by Earth-based sensors to back up the spaced-based sensors.

- Maximize Nuclear Survivability: Requires dispersal of hardened missile silos within a circular area of at least one hundred kilometers diameter.

- Minimize Base Vulnerability: Requires maximizing separation between bases to deter pre-emptive strikes and reduce vulnerability of transearth strike forces to interception.

- Minimize Transportation Costs: Requires selection of base zone locations to minimize fuel cost for both supply shuttles which utilize parking orbits and strike forces which require direct-ascent trajectories. (This issue will be difficult to resolve, because if both strike forces were to take advantage of the Moon's orbital velocity, then locations near the lunar poles will be favored. This induces a strategic problem, because the energy requirements for striking Northern-hemisphere Earth targets will differ between the polar bases.).

- Avoid Base Overflight: Requires designing supply shuttle parking orbits not to overfly opposing base.

- Minimize Communication Costs : Requires designing base communications on the near-side of the Moon. Far-side Moon bases require more communication system elements because its orientation with the Earth does not meet full-time line-of-sight communications link requirements.

C. SPACE SUPPORT ACTIVITIES

A large variety of space support programs can be served by SPI. For example, lunar-based astronomy would be free of restriction imposed by Earth's atmosphere. A lunar base could be used as a staging platform for a manned expedition to Mars, taking advantage of a smaller round trip propulsion requirement for liftoff from the Moon, rather than from the Earth. The servicing of geostationary satellites could be accomplished with SPI resources. Transport of concentrated nuclear waste to a safe dumping site in the Moon would be unhampered by groundwater problems as on the Earth (Rosen, 1981). SPI resources could be used for debris removal from low-Earth orbits. This is an increasingly serious hazard to space missions. Also, safe disposal of space junk or dead satellites in higher orbits could be an SPI priority.

Using the SPI surveillance network, the nuclear arsenal of the Moon can be used to deflect Earth threatening asteroids or comets. Collision with a ten-kilometer diameter asteroid has the explosive force of ten million one-megaton nuclear bombs. With an explosion of such magnitude, the impact dust cloud will plunge the world into a deep freeze similar to "nuclear winter" but much more devastating. Fossil records indicate that such an asteroid could account for extinction of the dinosaurs and cause for the Ice Age (Malove, 1985).

D. SUMMARY

The Strategic Peace Initiative Concept is a revolutionary plan for strategic nuclear deterrence. It is a plan which provides double value; a solution to accidental global

nuclear war by increasing reaction time to at least two days, and promotion of peaceful uses of space by developing permanently manned lunar bases and their infrastructures.

The uniqueness of SPI is that it requires bilateral treaty agreement between the United States and the Soviet Union. Although the politics of such a treaty requires extensive thought, verification of the treaty can be accomplished. By looking to the future of peace, perhaps we can be at peace with ourselves and with future generations.

IV. CONCLUSIONS AND RECOMMENDATIONS

This final chapter summarizes the findings if this thesis. It also recommends research for further work concerning this topic.

A. FINDINGS

This thesis utilizes the selection of an efficient and operationally meaningful set of inputs and terminal parameters, to lead to a transearth or translunar trajectory targeting solution, which rapidly converges in an iteration process.

B. RECOMMENDATIONS

This thesis lays the groundwork for an accurate patched conic method of designing fast transearth trajectories. Recommendations include the finalization of the patched conic approximation and the iterated impact point for the Moon-centered conic trajectory. A development of the SPI concept to include a cost/mission analysis would also be very useful.

APPENDIX A. PATCHED-CONIC BASED ON INERTIAL FRAMES

The patched conic trajectory can be formulated in terms of Earth-centered and Moon-centered coordinate frames which are inertial, being attached to a fictitious Earth and Moon which travel at constant velocity with respect to the barycentric frame. The Earth-centered conic trajectory approximation implies the lunar gravitational force is ignored so that the Earth is not accelerating with respect to the barycentric frame. Consequently, after the instant the spacecraft is launched, the center of gravitational force, the Earth, is replaced by a fictitious Earth which is imagined as moving in a straight line at constant velocity in the barycentric frame. In the meantime, the massless Moon continues on its circular orbit in the barycentric frame. In terms of the translating, Earth-centered frame, the lunar orbit appears as a compound curve, rather than a circle. This fact must be taken into account in targeting the conic trajectory for lunar intercept or crossing of the lunar-sphere-of-influence. The same considerations apply to the Moon-centered conic phase where the Moon is approximated by a fictitious Moon moving in a straight line at constant velocity in the barycentric frame. For greatest accuracy, the line is taken tangent to the Moon's circular orbit in the barycentric frame at the time of intercept. This requires an iterative solution. (Wadsworth, 1991)

APPENDIX B. LUNAR TRAJECTORY PROGRAM

INPUT PARAMETERS:

S/C declination at earth launch point

(input in degrees)

$\delta_E := \frac{\pi}{180} \cdot 28.3$

$\delta_E = 0.493928178314395$

radians

true anomaly of s/c at moon intercept

(input in degrees)

$f_M := \frac{\pi}{180} \cdot 164$

$f_M = 2.8623399732707$

radians

inclination of the moon orbit plane

(input in degrees:

$28.5 > i_M > 18.5$)

$i_M := \frac{\pi}{180} \cdot 20$

$i_M = 0.349065850398866$

radians

eccentricity of the translunar trajectory

($eccen >= eccen_{min}$)

$eccen := 1.01$

geocentric altitude of launch point

(altitude in kilometers)

$h_E := 1$

Inner sweep angle at moon intercept

(input in degrees)

$$\phi_M := \frac{\pi}{180}\cdot 170$$

$\phi_M = 2.96705972839036$
radians

posigrade or retrograde orbit determination

posigrade = +1

retrograde = −1

$\alpha_{E\_OBT} := 1$

$\delta_{M\_off} := 0$ (not iterated in program)

INTERMEDIATE OUTPUT:

$$\delta_M := \mathrm{asin}\left[\sin\left[i_M\right]\cdot\sin\left[\phi_M\right]\right]$$

$\delta_M = 0.059426145347819$

(spacecraft declination at moon intercept)

$$\alpha_{M1} := \left[\frac{\cos\left[\phi_M\right]}{\cos\left[\delta_M\right]}\right]$$

$\alpha_{M1} = -0.986549223316463$

$$\alpha_{M2} := \left[\frac{\left[\cos\left[i_M\right]\cdot\sin\left[\phi_M\right]\right]}{\cos\left[\delta_M\right]}\right]$$

$\alpha_{M2} = 0.163464460888851$

$$\alpha_M := 2\cdot\mathrm{atan}\left[\frac{\left[1-\alpha_{M1}\right]}{\alpha_{M2}}\right]$$

$\alpha_M = 2.97739131857372$

INPUT CONSTANTS:

$\rho_{EM} := 384400$ km : earth-moon mean distance

$r_E := 6378$ km : earth mean radius

$r_M := 1738$ km : moon mean radius

$\omega_E := 7.292 \cdot 10^{-5}$ rad/sec : earth angular rotation rate

$\omega_M := 1.525 \cdot 10^{-4} \cdot \frac{\pi}{180}$

$\omega_M = 2.661627109291352 \cdot 10^{-6}$ rad/sec :

moon mean orbital rotational rate

$k_E := 398600$ km^2/sec^3 : earth gravitational constant

$k_M := 4903$ km^2/sec^3 : moon gravitational constant

OUTPUT VARIABLES and FORMULAS:

$\omega_M := -1 \cdot \omega_M$ (due to the moon moving backwards in its orbit for the translunar trajectory)

$R_{E1} := r_E + h_E$

$R_{E1} = 6.379 \cdot 10^3$ km : (s/c radial distance at earth launch)

$R_{M1} := \rho_{EM}$

$R_{M1} = 3.844 \cdot 10^5$ km : (s/c radial distance at moon intercept)

$r := \frac{R_{M1}}{R_{E1}}$ (radial distance ratio)

$r = 60.26022887599937$

$f_E := \mathrm{acos}\left[\frac{r \cdot [1 + \mathrm{eccen} \cdot \cos[f_M]] - 1}{\mathrm{eccen}}\right]$

$f_E = 0.726288833059664$

$f_{EM} := f_M - f_E$

$f_{EM} = 2.136051140211037$

$\mathrm{sign}_{fEM} := \sin[f_{EM}]$ NOTE: for S_{fem}

$\mathrm{sign}_{fEM} = 0.84445214187218$ if sign is positive then use +1

if sign is negative use −1

$S_{fem} := 1$

$\alpha_{EM1} := \frac{[\cos[f_{EM}] - \sin[\delta_E] \cdot \sin[\delta_M]]}{[\cos[\delta_E] \cdot \cos[\delta_M]]}$

$\alpha_{EM1} = -0.641452764362017$

$$\alpha_{EM2} := \sqrt{1-\alpha_{EM1}^{2}} \cdot S_{fem} \cdot \alpha_{E\_OBT}$$

$$\alpha_{EM2} = 0.767162532383019$$

$$\alpha_{EM} := 2 \cdot \mathrm{atan}\left[\frac{\left[1-\alpha_{EM1}\right]}{\alpha_{EM2}}\right]$$

NOTE :

if $\alpha_{EM1} = -1$ then α_{EM} is $\pi \cdot \alpha_{E\_OBT}$

if $\alpha_{EM1} = +1$ then α_{EM} is 0

$$\alpha_{EM} = 2.267186781944015$$

$$\alpha_{E} := \alpha_{M} - \alpha_{EM}$$

$\alpha_{E} = 0.710204536629705$ (s/c right ascension at earth launch)

$$\delta_{E1} := -\pi + f_{M} - \delta_{M}$$

$$\delta_{E1} = -0.338678825666912$$

$$\delta_{E2} := \pi - f_{M} - \delta_{M}$$

$$\delta_{E2} = 0.219826534971273$$

$$f_{M\_MAX1} := \sin\left[\delta_{E}\right] \cdot \sin\left[\delta_{M}\right] + \cos\left[\alpha_{EM}\right] \cdot \cos\left[\delta_{E}\right] \cdot \cos\left[\delta_{M}\right]$$

$$f_{M\_MAX1} = -0.53563101113312$$

$$f_{M\_MAX2} := \sqrt{\left[1 - f_{M\_MAX1}^{2}\right]}$$

$$f_{M\_MAX2} = 0.84445214187218$$

$$f_{M\_MAX} := 2 \cdot \mathrm{atan}\left[\frac{\left[1 - f_{M\_MAX1}\right]}{f_{M\_MAX2}}\right]$$

$$f_{M\_MAX} = 2.136051140211037$$

If: $\delta_{E1} < \delta_E < \delta_{E2}$

then $f_M < f_{M\_MAX}$ is constrained
otherwise use α_E
as calculated

$$\Delta\alpha := \mathrm{asin}\left[\tan\left[\delta_E\right] \cdot \tan\left[\delta_M\right] - \left[\frac{\cos\left[f_M\right]}{\left[\cos\left[\delta_E\right] \cdot \cos\left[\delta_M\right]\right]}\right]\right]$$

$$\Delta\alpha = 1.570796326794897 - 0.496321232060315i$$

Constraint:

$$\alpha E_{MIN} := \alpha_M - \frac{\pi}{2} - \Delta\alpha$$

$$\alpha E_{MIN} = -0.164201335016073 + 0.496321232060315i$$

α_E must be greater than αE_{MIN}

$$eccen_{min} := \left[\frac{1 - \left[\frac{1}{r}\right]}{\sqrt{1 - \left[\frac{2}{r}\right] \cdot \cos\left[f_{EM}\right] + \left[\frac{1}{r^2}\right]}}\right]$$

$$eccen_{min} = 0.974647175723671$$

$$energy_{min} := \frac{\left[-k_E \cdot \left[1 + eccen_{min}\right]\right]}{\left[2 \cdot R_{M1}\right]}$$ (= minimum specific energy at launch)

$$energy_{min} = -1.023795999276086$$

$$\beta_{E1} := \left[\frac{\sin\left[\delta_M\right] - \sin\left[\delta_E\right] \cdot \cos\left[f_{EM}\right]}{\cos\left[\delta_E\right] \cdot \sin\left[f_{EM}\right]}\right]$$

$$\beta_{E1} = 0.421410460300705$$

$$\beta_{E2} := \left[\frac{\cos\left[\delta_M\right] \cdot \sin\left[\alpha_{EM}\right]}{\sin\left[f_{EM}\right]}\right]$$

$$\beta_{E2} = 0.906870014913464$$

$$\beta_E := 2 \cdot \operatorname{atan}\left[\frac{1 - \beta_{E1}}{\beta_{E2}}\right]$$ flight path azimuth at launch:

$\pi > \beta > -\pi$

$$\beta_E = 1.135796261808675$$

$$i_1 := \cos\left[\delta_E\right] \cdot \sin\left[\beta_E\right]$$

$$i_1 = 0.798478510707822$$

$$i_2 := \sqrt{1 - {i_1}^2}$$

$$i_2 = 0.602023311789352$$

$$i := 2 \cdot \operatorname{atan}\left[\frac{1 - i_1}{i_2}\right]$$ (translunar trajectory inclination: $0 <= i <= \pi$)

$$i = 0.646032654504068$$

$$\alpha E_{\Omega 1} := \frac{\beta_{E1}}{i_2}$$

$$\alpha E_{\Omega 1} = 0.699990269559789$$

$$\alpha E_{\Omega 2} := \frac{\tan\left[\delta_E\right]}{\tan(i)}$$

$$\alpha E_{\Omega 2} = 0.714152380463452$$

$$\alpha E_{\Omega} := 2 \cdot \operatorname{atan}\left[\frac{1 - \alpha E_{\Omega 1}}{\alpha E_{\Omega 2}}\right]$$

$$\alpha E_{\Omega} = 0.795412455434794$$

$$\Omega := \alpha_E - \alpha E_{\Omega}$$ (translunar trajectory longitude-of-ascending-node)

$$\Omega = -0.085207918805089$$

$$C_{E1} := \cos\left[\delta_E\right] \cdot \cos\left[\alpha E_{\Omega}\right]$$

$$C_{E1} = 0.616325580024169$$

$$C_{E2} := \left[\frac{\sin[\delta_E]}{i_2}\right]$$

$$C_{E2} = 0.787491447196649$$

$$C_E := 2 \cdot \operatorname{atan}\left[\frac{1 - C_{E1}}{C_{E2}}\right]$$

$$C_E = 0.906728172342672$$ (sweep angle at launch: $2\pi > C_E >= 0$)

$$R_p := \left[\frac{R_{E1} \cdot [1 + eccen \cdot \cos[f_E]]}{1 + eccen}\right]$$ (perigee distance)

$$R_p = 5.57010652841956 \cdot 10^3$$

$$V_p := \sqrt{\frac{k_E \cdot (1 + eccen)}{R_p}}$$ (perigee velocity)

$$V_p = 11.99319583652338$$

$$V_E := \sqrt{\left[\frac{2 \cdot k_E}{R_{E1}}\right] - \left[\frac{(1 - eccen) \cdot k_E}{R_p}\right]}$$ (s/c velocity at launch)

$$V_E = 11.21107362995628$$

$$R_a := \frac{R_p \cdot (1 - eccen)}{1 - eccen}$$ (apogee distance; valid only for eccen < 1

ignore for eccen > = 1)

$$R_a = 5.57010652841956 \cdot 10^3$$

$$f_{asy} := \pi - \operatorname{atan}\left[\sqrt{eccen^2 - 1}\right]$$ (hyperbolic asymptotic true anomaly

half-angle ; valid only for eccen> 1

$$f_{asy} = 3.000756780023376$$ ignore for eccen < = 1)

$$D := 2 \cdot f_{asy} - \pi$$ (deflection angle)

$$D = 2.859920906456958$$

$$d_{asy} := R_p \cdot \left[1 + \left[\frac{1}{eccen}\right]\right]$$ (directrix-to-focus distance or axial offset

of asymptotic from focus ; valid only for

eccen > 1 ignore for eccen < = 1)

$$d_{asy} = 1.108506348725081 \cdot 10^4$$

$$\gamma_{E1} := \frac{R_p \cdot V_p}{R_{E1} \cdot V_E}$$

$$\gamma_{E1} = 0.934111286367402$$

$$\gamma_{E2} := \frac{eccen \cdot \gamma_{E1} \cdot \sin\left[f_E\right]}{1 + eccen \cdot \cos\left[f_E\right]}$$

$$\gamma_{E2} = 0.356981938872315$$

$$\gamma_E := 2 \cdot \operatorname{atan}\left[\frac{1-\gamma_{E1}}{\gamma_{E2}}\right]$$

(flight path angle at launch;

$\frac{\pi}{2} > \gamma_E > \frac{-\pi}{2}$

$$\gamma_E = 0.365034946775507$$

$$V_M := \sqrt{\frac{2 \cdot k_E}{R_{M1}} - \left[\frac{(1-\mathrm{eccen}) \cdot k_E}{R_p}\right]}$$

(s/c velocity at moon intercept)

$$V_M = 1.670175762283258$$

$$\gamma_{M1} := \frac{R_p \cdot V_p}{R_{M1} \cdot V_M}$$

$$\gamma_{M1} = 0.104052584194718$$

$$f_{M1} := \cos\left[f_M\right]$$

$$f_{M2} := \sin\left[f_M\right]$$

$$\gamma_{M2} := \frac{\mathrm{eccen} \cdot \gamma_{M1} \cdot f_{M2}}{1+\left[\mathrm{eccen} \cdot f_{M1}\right]}$$

$$\gamma_{M2} = 0.99457179716821$$

$$\gamma_M := 2 \cdot \operatorname{atan}\left[\frac{1-\gamma_{M1}}{\gamma_{M2}}\right]$$

(flight path angle at moon intercept)

$$\gamma_M = 1.466555060009538$$

$$\beta_{M1} := i_2 \cdot \left[\cos\left[\alpha_M - \Omega\right]\right]$$

$$\beta_{M1} = -0.600145987662734$$

$$\beta_{M2} := \left[\frac{i_1}{\cos\left[\delta_M\right]}\right]$$

$$\beta_{M2} = 0.799890488437212$$

$$\beta_M := 2 \cdot \operatorname{atan}\left[\frac{1-\beta_{M1}}{\beta_{M2}}\right]$$ (flight path azimuth at moon intercept)

$$\beta_M = 2.214479932657055$$

$$X_M := R_{M1} \cdot \cos\left[\delta_M\right] \cdot \cos\left[\alpha_M\right]$$ (RECTANGULAR COORDINATES OF THE MOON AT POINT-MOON INTERCEPT)

$$X_M = -3.785601002578928 \cdot 10^5$$

$$Y_M := R_{M1} \cdot \cos\left[\delta_M\right] \cdot \sin\left[\alpha_M\right]$$

$$Y_M = 6.272482025241596 \cdot 10^4$$

$$Z_M := R_{M1} \cdot \sin\left[\delta_M\right]$$

$$Z_M = 2.282996752157727 \cdot 10^4$$

$$\omega := C_E - f_E$$ (translunar trajectory argument-of-perigee)

$$\omega = 0.180439339283008$$

$$C_M := f_M + \omega$$

$C_M = 3.042779312553709$ (sweep angle at moon intercept)

$$X_{M'} := \left[\left[\frac{V_M}{\cos\left[\delta_M\right]}\right]\cdot\left[\left[\gamma_{M2} - \sin\left[\delta_M\right]\cdot i_2\cdot\cos\left[C_M - \gamma_M\right]\right]\cdot\cos\left[\alpha_M - \Omega\right]\right] \ldots \atop + -1\cdot\left[i_1\cdot\gamma_{M1}\cdot\sin\left[\alpha_M - \Omega\right]\right]\right]$$

$X_{M'} = -1.665737958076395$

$$Y_{M'} := \left[\left[\frac{V_M}{\cos\left[\delta_M\right]}\right]\cdot\left[\left[\gamma_{M2} - \sin\left[\delta_M\right]\cdot i_2\cdot\cos\left[C_M - \gamma_M\right]\right]\cdot\sin\left[\alpha_M - \Omega\right]\right] \ldots \atop + \left[i_1\cdot\gamma_{M1}\cdot\cos\left[\alpha_M - \Omega\right]\right]\right]$$

$Y_{M'} = 0.048513058379955$

$$Z_{M'} := V_M\cdot i_2\cdot\cos\left[C_M - \gamma_M\right]$$

$Z_{M'} = -0.00545766973131$

$$R_{M'} := \frac{\left[X_M\cdot X_{M'}\right] + \left[Y_M\cdot Y_{M'}\right] + \left[Z_M\cdot Z_{M'}\right]}{R_{M1}}$$

(radial velocity at point-moon intercept)

$R_{M'} = 1.648023680688986$

$$T_p := \frac{2 \cdot \pi \cdot R_p^{1.5}}{k_E^{.5}}$$ (circular orbit period for radius R_p)

$$T_p = 4.137199589247244 \cdot 10^3$$

$$f_{E1} := \frac{1 - \cos\left[f_E\right]}{\sin\left[f_E\right]}$$

$$f_{E1} = 0.37999703209726$$

$$f_{M1} := \frac{1 - \cos\left[f_M\right]}{\sin\left[f_M\right]}$$

$$f_{M1} = 7.115369722384208$$

$$E_{h1} := \ln\left[\frac{1 + \sqrt{\frac{eccen - 1}{1 + eccen}} \cdot f_{E1}}{1 - \sqrt{\frac{eccen - 1}{1 + eccen}} \cdot f_{E1}}\right]$$

$$E_{h1} = 0.05361869041727$$

$$E_{h2} := \ln\left[\frac{1 + \sqrt{\frac{eccen - 1}{1 + eccen}} \cdot f_{M1}}{1 - \sqrt{\frac{eccen - 1}{1 + eccen}} \cdot f_{M1}}\right]$$

$$E_{h2} = 1.103630544045282$$

$$E_{sh1} := \left[\left[1 - \sqrt{\frac{eccen - 1}{1 + eccen}} \cdot f_{E1} \right]^{-1} - \left[1 + \sqrt{\frac{eccen - 1}{1 + eccen}} \cdot f_{E1} \right]^{-1} \right]$$

$$E_{sh1} = 0.053644386077804$$

$$E_{sh2} := \left[\left[1 - \sqrt{\frac{eccen - 1}{1 + eccen}} \cdot f_{M1} \right]^{-1} - \left[1 + \sqrt{\frac{eccen - 1}{1 + eccen}} \cdot f_{M1} \right]^{-1} \right]$$

$$E_{sh2} = 1.34171391602585$$

$$t_{E1} := \frac{\left[eccen \cdot E_{sh1} \right] - E_{h1}}{2 \cdot \pi \cdot (eccen - 1)^{1.5}}$$

$$t_{E1} = 0.089467283524084$$

$$t_{M1} := \frac{\left[eccen \cdot E_{sh2} \right] - E_{h2}}{2 \cdot \pi \cdot (eccen - 1)^{1.5}}$$

$$t_{M1} = 40.02754953820076$$

$t_E := T_p \cdot t_{E1}$ (earth launch flight time relative to perigee passage)

$$t_E = 370.144008646907$$

$t_M := T_p \cdot t_{M1}$ (point-moon intercept flight time relative to perigee passage)

$t_M = 1.656019615080179 \cdot 10^5$

$t_{EM} := t_M - t_E$ (point-moon translunar trajectory flight time)

$t_{EM} = 1.65231817499371 \cdot 10^5$

INPUTS:

$R_0 := 382986.2976625$ (any radial distance)

OUTPUTS:

$$V_0 := \sqrt{\frac{[2 \cdot k_E]}{R_0} - \frac{[(1 - eccen) \cdot k_E]}{R_p}}$$

$V_0 = 1.67246593 80983$

$$R_x := \frac{[R_p \cdot (1 + eccen)]}{R_0}$$

$R_x = 0.029233197611653$

$$f_0 := 2 \cdot atan\left[\left[\frac{[eccen + 1 - R_x]}{[eccen - 1 + R_x]}\right]^{.5}\right]$$

$f_0 = 2.861954051456415$

$$C_0 := \omega + f_0$$

$$C_0 = 3.042393390739423$$

$$\delta_0 := \mathrm{asin}\left[\sin(i)\cdot\sin\left[C_0\right]\right]$$

$$\delta_0 = 0.059657751937721$$

$$\Delta\alpha_{M1} := \frac{\left[\tan\left[\delta_M\right]\right]}{\left[\sin(i)\cdot\tan\left[C_M\right]\right]}$$

$$\Delta\alpha_{M1} = -0.996881642139325$$

$$\Delta\alpha_{M2} := \frac{\tan\left[\delta_M\right]}{\tan(i)}$$

$$\Delta\alpha_{M2} = 0.078911289214964$$

$$\Delta\alpha_M := 2\cdot\mathrm{atan}\left[\frac{\left[1-\Delta\alpha_{M1}\right]}{\Delta\alpha_{M2}}\right]$$

$$\Delta\alpha_M = 3.062599237378809$$

$$\Delta\alpha_1 := \frac{\tan\left[\delta_0\right]}{\left[\sin(i)\cdot\tan\left[C_0\right]\right]}$$

$$\Delta\alpha_1 = -0.996857191524672$$

$$\Delta\alpha_2 := \frac{\tan\left[\delta_0\right]}{\tan(i)}$$

$$\Delta\alpha_2 = 0.079219566431173$$

$$\Delta\alpha_0 := 2\cdot\operatorname{atan}\left[\frac{\left[1-\Delta\alpha_1\right]}{\Delta\alpha_2}\right]$$

$$\Delta\alpha_0 = 3.062289992046826$$

$$\alpha_s := \alpha_M - \Delta\alpha_M$$

$$\alpha_s = -0.085207918805089$$

$$\alpha_0 := \Delta\alpha_0 + \alpha_s$$

$$\alpha_0 = 2.977082073241737$$

$$\gamma_1 := \frac{\left[R_p\cdot V_p\right]}{\left[R_0\cdot V_0\right]}$$

$$\gamma_1 = 0.104293660037493$$

$$\gamma_2 := \frac{\left[\text{eccen}\cdot\gamma_1\cdot\sin\left[f_0\right]\right]}{\left[1+\text{eccen}\cdot\cos\left[f_0\right]\right]}$$

$$\gamma_2 = 0.994546546158593$$

$$\gamma_0 := 2\cdot\mathrm{atan}\left[\frac{[1-\gamma_1]}{\gamma_2}\right]$$

$$\gamma_0 = 1.466312665340157$$

$$\beta_1 := i_2\cdot\cos[\alpha_0-\Omega]$$

$$\beta_1 = -0.600131267822715$$

$$\beta_2 := \left[\frac{i_1}{\cos[\delta_0]}\right]$$

$$\beta_2 = 0.799901532303508$$

$$\beta_0 := 2\cdot\mathrm{atan}\left[\frac{[1-\beta_1]}{\beta_2}\right]$$

$$\beta_0 = 2.214461530464985$$

$$X_0 := R_0\cdot\cos[\delta_0]\cdot\cos[\alpha_0]$$

(RECTANGULAR COORDINATES OF

$$X_0 = -3.771433240671579\cdot10^5$$

THE SPACECRAFT ON ITS TRAJECTORY)

$$Y_0 := R_0\cdot\cos[\delta_0]\cdot\sin[\alpha_0]$$

$$Y_0 = 6.260990806241751\cdot10^4$$

$$Z_0 := R_0\cdot\sin[\delta_0]$$

$$Z_0 = 2.283455104046146\cdot10^4$$

$$f_{M0} := \frac{\left[1-\cos\left[f_0\right]\right]}{\sin\left[f_0\right]}$$

$$f_{M0} = 7.105421102050813$$

$$E_{h3} := \ln\left[\frac{\left[1+\left[\sqrt{\frac{(eccen-1)}{(1+eccen)}}\right]\cdot f_{M0}\right]}{\left[1-\left[\sqrt{\frac{(eccen-1)}{(1+eccen)}}\right]\cdot f_{M0}\right]}\right]$$

$$E_{h3} = 1.101755458701495$$

$$E_{sh3} := \left[\left[1-\left[\sqrt{\frac{(eccen-1)}{(1+eccen)}}\right]\cdot f_{M0}\right]^{-1} - \left[1+\left[\sqrt{\frac{(eccen-1)}{(1+eccen)}}\right]\cdot f_{M0}\right]^{-1}\right]$$

$$E_{sh3} = 1.338578545032142$$

$$t_{M0} := \frac{\left[\left[eccen\cdot E_{sh3}\right]-E_{h3}\right]}{\left[2\cdot\pi\cdot(eccen-1)^{1.5}\right]}$$

$$t_{M0} = 39.8219787493874$$

$$t_0 := T_p\cdot t_{M0}$$

$$t_0 = 1.647514741249780\cdot 10^5$$

$t_{off} := 0$ (time offset input)

$\Delta\tau_0 := t_M - t_0 + t_{off}$

$\Delta\tau_0 = 850.4873830398719$

$\phi_{M1} := \cos\left[\alpha_M\right] \cdot \cos\left[\delta_M\right]$

$\phi_{M1} = -0.984807753012208$

$$\phi_{M2} := \frac{\sin\left[\delta_M\right]}{\sin\left[i_M\right]}$$

$\phi_{M2} = 0.17364817766693$

$$\phi_M := 2 \cdot \operatorname{atan}\left[\frac{\left[1 - \phi_{M1}\right]}{\phi_{M2}}\right]$$

$\phi_M = 2.96705972839036$

$\phi_{M0} := \phi_M - \omega_M \cdot \Delta\tau_0$

$\phi_{M0} = 2.969323408665169$

$X_{M0} := \rho_{EM} \cdot \cos[\phi_{M0}]$ (RECTANGULAR COORDINATES OF THE MOON AT TIME t_0)

$X_{M0} = -3.787102316833942 \cdot 10^5$

$Y_{M0} := \rho_{EM} \cdot \cos[i_M] \cdot \sin[\phi_{M0}]$

$Y_{M0} = 6.191940094679928 \cdot 10^4$

$Z_{M0} := \rho_{EM} \cdot \sin[i_M] \cdot \sin[\phi_{M0}]$

$Z_{M0} = 2.253681886822944 \cdot 10^4$

$$D_{M0} := \sqrt{\left[\left[X_0 - X_{M0}\right]^2 + \left[Y_0 - Y_{M0}\right]^2 + \left[Z_0 - Z_{M0}\right]^2\right]}$$

DISTANCE BETWEEN THE SPACECRAFT AND THE LUNAR CENTER AT TIME t_0

$D_{M0} = 1.738000000264939 \cdot 10^3$ km

THE GOAL IS TO DRIVE THIS DISTANCE TO EQUAL THE RADIUS OF THE MOON FOR AN IMPACT ON THE SURFACE OF THE MOON

$$\text{error} := D_{M0} - r_M$$

$$\text{error} = 2.649394446052611 \cdot 10^{-7}$$

$$\Delta_{\rho 0} := \frac{\left[\left[X_{M0} - X_0\right] \cdot X_0 + \left[Y_{M0} - Y_0\right] \cdot Y_0 + \left[Z_{M0} - Z_0\right] \cdot Z_0\right]}{\left[R_0 \cdot D_{M0}\right]}$$

$\Delta_{\rho 0} = 0.81263979792074$ ($=$ $\cos[\lambda_N]$)

$$R_{next} := R_0 + D_{M0} \cdot \Delta_{\rho 0}$$

$R_{next} = 3.843986656315015 \cdot 10^5$ (number to use for next iteration of R_0)

$X := X_M - X_{M0}$

$X = 150.1314255014295$

$Y := Y_M - Y_{M0}$

$Y = 805.4193056166769$

$Z := Z_M - Z_{M0}$

$Z = 293.148653347831$

INPUTS:

$\phi := 0$ (EULER ANGLE INPUTS)

$\theta := i_M$

$\Psi_M := \phi_{M0}$

$\Psi_M = 2.969323408665169$

OUTPUTS:

(THE MATRIX STRUCTURE IS NUMBERED AS FOLLOWS:)

$$\begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}$$ (ROTATION AND TRANSLATION)

$A_1 := \cos\left[\Psi_M\right]\cdot\cos(\phi) - \cos(\theta)\cdot\sin(\phi)\cdot\sin\left[\Psi_M\right]$

$A_1 = -0.985198313432347$

$B_1 := -\sin\left[\Psi_M\right]\cdot\cos(\phi) - \cos(\theta)\cdot\sin(\phi)\cdot\cos\left[\Psi_M\right]$

$B_1 = -0.171418444777856$

$C_1 := \sin(\theta)\cdot\sin(\phi)$

$C_1 = 0$

$A_2 := \cos\left[\Psi_M\right]\cdot\sin(\phi) + \cos(\theta)\cdot\cos(\phi)\cdot\sin\left[\Psi_M\right]$

$A_2 = 0.161080647624348$

$B_2 := -\sin\left[\Psi_M\right]\cdot\sin(\phi) + \cos(\theta)\cdot\cos(\phi)\cdot\cos\left[\Psi_M\right]$

$B_2 = -0.925783585143099$

$C_2 := -\sin(\theta)\cdot\cos(\phi)$

$C_2 = -0.342020143325669$

$$A_3 := \sin(\theta)\cdot\sin\left[\Psi_M\right]$$

$$A_3 = 0.058628561051585$$

$$B_3 := \sin(\theta)\cdot\cos\left[\Psi_M\right]$$

$$B_3 = -0.336957668364338$$

$$C_3 := \cos(\theta)$$

$$C_3 = 0.939692620785908$$

$$A := \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}$$

$$B := \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$C := A\cdot B$$

$$C = \begin{bmatrix} -285.9729519600736 \\ -821.7234494824876 \\ 12.87940451383525 \end{bmatrix}$$

$C_0 = -285.9729519600736$

$C_1 = -821.7234494824876$

$C_2 = 12.87940451383525$

$X_{lun} := C_0$

$Y_{lun} := C_1$

$Z_{lun} := C_2$

$$\delta := \operatorname{atan}\left[\frac{Z_{lun}}{\sqrt{\left[X_{lun}^2 + Y_{lun}^2\right]}}\right]$$

$\delta = 0.014801756974441$

SELENOGRAPHIC LATITUDE FOR SURFACE

IMPACT ON THE MOON

$$\alpha := -\operatorname{acos}\left[\frac{X_{lun}}{\sqrt{\left[X_{lun}^2 + Y_{lun}^2\right]}}\right] + \pi$$

$\alpha = 1.235890035212915$

SELENOGRAPHIC LONGITUDE FOR SURFACE

IMPACT ON THE MOON

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